2.3. Performance Projections and Modeling

A series of calculations and simulations have been completed to assess the performance of the Recycler Ring. The Recycler is designed to accept antiprotons from the Accumulator and from the Main Injector at 8.9 GeV/c. The antiproton beam is required to be stored in the Recycler for many hours under the influence of a stochastic cooling systems with cooling times of approximately 2 hours. The long cooling time precludes the possibility of a complete tracking simulation of the entire accumulation scenario. Instead we rely on a combination of semi-analytic calculations and tracking of particles over a range of 10^5 to 10^6 turns (1-10 seconds of real time). In general the strategy is to use the calculations to suggest criteria for alignment and field quality requirements and then utilize tracking simulations to observe the survivability of particles in the model Recycler as a function of betatron oscillation amplitude and momentum offset.

Tracking calculations are performed using the thin element tracking program TEAPOT [L. Schachinger and R. Talman, Particle Accelerators 22, 1987]. Because of our inability to track particles for more than a few seconds we require survivability for particles with oscillation amplitudes corresponding to at least 70 π mmmr (normalized) over the full ±0.3% momentum aperture of the Recycler. This oscillation amplitude corresponds to the vertical physical vacuum chamber aperture and provides a 50% margin relative to the specified 40 π mmmr dynamic aperture. As Recycler prototype magnets become available it is assumed that these tracking calculations will continue utilizing measured field characteristics.

In this section we describe the sensitivity of the Recycler lattice (RRV10) to a variety of alignment and field non-uniformities that manifest themselves as closed orbit errors, betatron function errors, horizontal-vertical coupling, and finite dynamic aperture. The description of the Recycler lattice includes higher magnetic multipoles in combined function and quadrupole magnets, and both magnet and beam position monitor alignment errors. As described in section 2.2 this lattice is based on a ~90° phase advance cell, with a maximum beta function of 55 m and a maximum dispersion of about 2 m. The vertical dispersion in the Recycler is essentially zero and the nominal tune is (Q_x,Q_y)=(25.425, 24.415). The lattice functions for the Recycler ring are shown in figure 2.2.6.

2.3.1 Recycler error matrix and expected alignment tolerances

The Recycler Ring utilizes a total of 344 combined function (a.k.a. gradient) and 86 quadrupole magnets to provide bending and optical focusing. Construction of the ring with the expected optical characteristics depends upon the production and alignment of magnets with a high degree of accuracy. In addition, since these elements are all implemented as permanent magnets, the absolute strengths need to be well controlled.

In the presence of alignment and magnetic field errors the Recycler lattice will exhibit a number of effects including closed orbit distortion, variations from the nominal tune, beta function distortions, horizontal-vertical tune splits due to coupling, and limited dynamic aperture. Magnetic field errors will be of two generic types; systematic and random. Systematic errors are reflected in a variation of the average value for a type of magnet relative to a desired value, while random errors refer to the width of the distribution around the mean. The effects of systematic and random errors are different and so we treat them separately here.

Two primary magnet classes are considered. The first class is represented by the long (4.3 m) and the short (2.8 m) gradient magnets, while the second class is represented by the total of all quadrupole magnets. The short gradient magnets differ from the long in total integrated bending strength, nominally 2/3 of the long magnets, in their gradient-to-dipole ratio, and in the absence of a built-in sextupole component. The quadrupole magnets come in 10 flavors representing a variety of strengths. Magnet strength requirements are summarized in section 3.1.

Orbit, tune, beta function, and coupling sensitivities of the Recycler lattice have been calculated for systematic and random magnetic and alignment errors for each class of magnet. Results are displayed in table 2.3.1 for systematic errors and in table 2.3.2 for random errors. Systematic sensitivities are calculated via direct application to the Recycler design lattice while random sensitivities are calculated via statistical expressions as given below. Examples of specific orbit and lattice variations can be found in sections 2.3.2 and 2.3.3.

Table 2.3.1: Sensitivity of the Recycler lattice to systematic errors. Root-mean-square orbit and beta function distortions, as well as tune shifts and global coupling strength are given for the indicated systematic errors. All effects are linear in the assumed underlying errors.

	Orbit (σ_x)	ΔQ_x	ΔQ_y	$\Delta\beta_x/\beta_x$ (rms)	$\Delta\beta_y/\beta_y$ (rms)	ΔQ_{min}
Gradient magnets						
integrated dipole strength $\Delta BL/BL=.0001^*$	0.3 mm					
integrated gradient $\Delta B'L/BL=.0001$/inch		.037	-.037	38x10^{-4}	98x10^{-4}	
skew quadrupole $\Delta B'_sL/BL=.0001$/inch						.0218
Quadrupole magnets						
integrated strength $\Delta B'L/B'L=.0001$		.0004	-.0004	2.9x10^{-4}	5.1x10^{-4}	

* Orbit entry represents a systematic shift between the short magnet and the target of 2/3 the long magnet strength.

In table 2.3.1 it is assumed that the energy at which antiprotons are delivered to the Recycler will be adjusted to correspond to the average bending field in the long gradient magnets. Thus the only entry for a closed orbit error is associated with a mismatch between the strength of the short magnet and 2/3 the strength of the long magnet. As seen from the table a 10^{-4} systematic mismatch of these two magnets results in a 0.3 mm rms orbit distortion. The beta function distortion caused by gradient errors shows relatively suppressed sensitivity in the gradient magnets. This is because of the natural suppression to systematic gradient errors provided by the ~90° cells making up the Recycler. The sensitivity of the tune split to a systematic skew quadrupole has been reduced by providing a one unit tune split between the horizontal and vertical tunes. It should be noted that table 2.3.1 makes no provision for any sort of systematic magnet misalignment.

Table 2.3.2 shows the sensitivity of the lattice to both random magnetic and alignment errors. Closed orbit errors are evaluated utilizing the expression:

$$\frac{\sigma_x^2(s)}{\beta_x(s)} = \frac{1}{8\sin^2(\pi\nu_x)}\sum_i \beta_{x_i}(\sigma_{\theta_i})^2 \quad , \quad (2.3.1)$$

where σ_{θ_i} is the rms bending angle error through the i^{th} element. For a dipole magnet σ_{θ_i} is $\sigma_{BL}/(B\rho)$ (horizontally) or $\sigma_\phi \times BL/(B\rho)$ where ϕ is the roll angle (vertically). For any magnet containing a field gradient, σ_{θ_i} is $\sigma_d \times B'L/(B\rho)$ where d is the transverse displacement. The interpretation of the numbers in table 2.3.2 is that at any given point in the ring, there would be a ~2/3 chance of observing an orbit distortion of $<(\sigma_x/\sqrt{\beta})\times\sqrt{\beta}(s)$ if magnetic elements were fabricated and aligned to the noted tolerances. A total rms orbit distortion at any point due to a different set of errors can be calculated by noting that distortions are linear in the corresponding errors and that all distortions add in quadrature. For example, a magnet-to-magnet (rms) strength variation of 5×10^{-4}, accompanied by an rms displacement error of 0.25 mm and an rms roll angle of 0.5 mrad would produce an rms orbit distortion of 5.8 mm horizontally and 6.1 mm vertically (β_{av}=33 m).

Beta function distortions are calculated using the expression:

$$\sigma^2_{\frac{\Delta\beta}{\beta}} = \frac{1}{8\sin^2(2\pi\nu)}\sum_i (\sigma_{kl})_i^2\beta_i^2 \quad , \quad (2.3.2)$$

where σ_{kl} is the integrated gradient error, or the transverse displacement times the integrated sextupole strength in the case of the long gradient magnets. The minimum tune split is calculated as

$$(\Delta\nu_{min})^2 = \frac{1}{2\pi^2}\sum_i (\sigma_{k_s l})_i^2\beta_{x_i}\beta_{y_i} \quad , \quad (2.3.3)$$

where $\sigma_{k_s l}$ is taken as σ_ϕ times the nominal integrated gradient. For any set of errors the expected rms beta distortion or minimum tune split can be calculated by scaling the numbers listed in the tables and then adding in quadrature. For example, an rms integrated gradient error of 1×10^{-4}/inch as referenced to the dipole field, accompanied by an rms 0.25 mm transverse offset would produce a distortion of 3.8% in both the horizontal and vertical beta functions. It should be noted through a comparison of table 2.3.2 with 2.3.1 that systematic (normal) gradient errors are more benign than random errors.

Table 2.3.2: Sensitivity of the Recycler lattice to random errors. Root-mean-square orbit and beta function distortions, as well as global coupling strength are given for the indicated random errors. All effects are linear in the assumed underlying errors and results add in quadrature

	Orbit Error ($\sigma_x/\sqrt{\beta_x}$)	Orbit Error ($\sigma_y/\sqrt{\beta_y}$)	$\Delta\beta_x/\beta_x$ (rms)	$\Delta\beta_y/\beta_y$ (rms)	ΔQ_{min}
Gradient magnets					
integrated dipole strength $\sigma_{BL}/BL=.0001$	$1.1\text{x}10^{-4}$				
integrated gradient $\sigma_{B'L}/BL=.0001/\text{inch}$			$386\text{x}10^{-4}$	$375\text{x}10^{-4}$	
skew quadrupole $\sigma_{B'sL}/BL=.0001/\text{inch}$					.0110
transverse displacement $\sigma_d=.00025$ m	$7.9\text{x}10^{-4}$	$8.4\text{x}10^{-4}$	$54\text{x}10^{-4}$	$84\text{x}10^{-4}$	
roll $\sigma_\phi=.0005$		$4.6\text{x}10^{-4}$			.0040
Quadrupole magnets					
integrated strength $\sigma_{B'L}/B'L=.0001$			$10.\text{x}10^{-4}$	$9.8\text{x}10^{-4}$	
transverse displacement $\sigma_d=.00025$ m	$2.7\text{x}10^{-4}$	$2.9\text{x}10^{-4}$			
roll $\sigma_\phi=.0005$					.0015

2.3.2 Closed orbit calculation and correction procedure

The Recycler contains 416 beam position monitors (BPMs), one horizontal and one vertical in each half cell. The closed orbit as observed on the BPM system is calculated for a random collection of alignment and bending strength errors as described in table 2.3.3. Orbit errors in the horizontal plane are generated by random transverse misalignments of the combined function and quadrupole magnets, and magnet-to-magnet bending strength variations, while in the vertical plane roll alignment errors replace strength variations as a contributor.

Table 2.3.3. Random misalignment errors used in Recycler orbit and tracking simulations.

Magnet Type	σ_{BL}/BL	σ_h (mm)	σ_v (mm)	σ_{roll} (mrad)
Gradient Magnet	.0005	0.25	0.25	0.50
Quadrupoles	-	0.25	0.25	0.50
Beam Position Monitors	-	0.25	0.25	-

Ten sets of Recycler closed orbits are generated using random distributions of strength and alignment errors with rms widths as given in table 2.3.3. Errors are generated in Gaussian distributions cut off at 3σ. The equality of the magnet-to-magnet and roll distribution widths implies that the vertical and horizontal orbits will be the same in a statistical sense. As a result only the horizontal orbit is studied here.

A histogram of the distribution of observed rms horizontal orbit distortions is given in figure 2.3.1. The figure shows that the most likely rms orbit distortion for tolerances given above lies in the 4-8 mm range. As can be deduced from table 2.3.2, magnet-to-magnet strength variation, transverse alignment, and roll errors all contribute approximately equally to the closed orbit distortion.

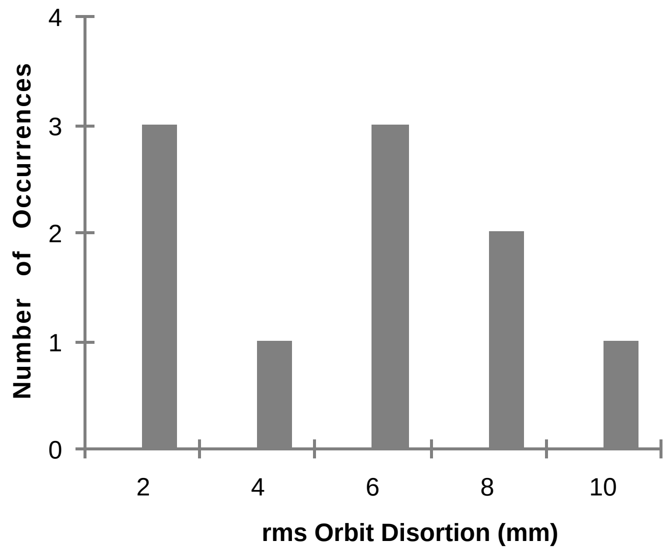

Figure 2.3.1: Probability distribution of expected uncorrected rms horizontal orbit distortion based on the alignment and strength errors given in table 2.3.3. Ten different ensembles of errors are calculated and entered into this histogram.

It will be desirable to correct the orbit to a residual distortion of under 1 mm. Because of the static nature of the Recycler it is possible to consider doing this by displacing the combined function magnets transversely in an appropriate manner. Several different strategies are possible for identifying a finite set of magnets that could and should be moved to correct the orbit. In the approach studied here a specific algorithm is applied to select a set of magnet moves that reduce the rms orbit distortion. This algorithm is believed to have originated in the CERN SPS and is implemented as follows:

1. For the closed orbit given, find the single magnetic element in the ring for which a move minimizes the rms orbit distortion. If an element has to be moved more than 5 mm in order to minimize the distortion that element is removed from consideration.

2. Find the second element that paired with the element identified in 1, is most effective in minimizing the rms orbit distortion. Again reject any element that requires a move of more than 5 mm to minimize the orbit distortion.

3. Repeat up to 15 times, always identifying the element to add to the previously defined list as most effective at minimizing the rms orbit distortion.

The effectiveness of the algorithm is demonstrated in table 2.3.4 and in figure 2.3.2. The table lists the rms and peak orbit distortions observed both before and after corrections for each of the ten generated orbits. In addition the maximum transverse magnet move is listed. The table indicates that in all cases the rms distortion can be reduced to less than 1 mm, with a peak distortion under 2.6 mm, by utilizing 15 suitably chosen magnet moves. In all instances the maximum magnet move is less than 3.3 mm. Figure 2.3.2 shows the entire uncorrected and corrected orbit for seed 8, the seed with the worst uncorrected orbit.

Table 2.3.4: Uncorrected and corrected orbit distortions for ten different randomly generated closed orbits. The column label "Δ_{max}" lists the maximum magnitude of the 15 magnet moves required to effect the correction.

Seed	Uncorrected		Corrected (15 moves)		
	σ_H (mm)	Peak (mm)	σ_H (mm)	Peak (mm)	Δ_{max} (mm)
1	2.87	6.85	0.65	2.61	2.54
2	5.69	13.45	0.66	2.17	2.23
3	7.83	17.57	0.65	2.04	2.80
4	2.69	6.35	0.52	1.66	3.28
5	7.55	17.27	0.75	2.04	2.72
6	2.49	6.02	0.73	2.08	3.06
7	6.06	11.01	0.56	1.52	2.89
8	9.00	16.40	0.73	1.79	2.69
9	5.67	11.33	0.77	1.84	2.82
10	3.69	8.87	0.59	1.77	2.06

2.3.3 Optical function errors

As discussed in section 2.3.1, combined function and quadrupole magnetic strength and alignment errors change the beta function around the Recycler from that of the ideal lattice. Figure 2.3.3 shows an example of the beta function distortion for a random distribution of gradient errors of $1\text{x}10^{-4}$/inch (rms, relative to the nominal dipole field) in the combined function magnets and $8\text{x}10^{-4}$ of the nominal gradient in the quadrupole magnets. The rms beta function deviation is a few percent and is dominated by the contribution from the long combined function magnets. The relatively large (15%) variation on the right hand side of figure 2.3.3 is caused by adjustments in the phase trombone at MI-60 required to maintain the tune at its nominal values in the presence of the gradient errors modeled.

No special provision are currently planned for correcting beta function distortions. Magnet performance specifications will be tailored to keep uncorrected distortions less than 2%.

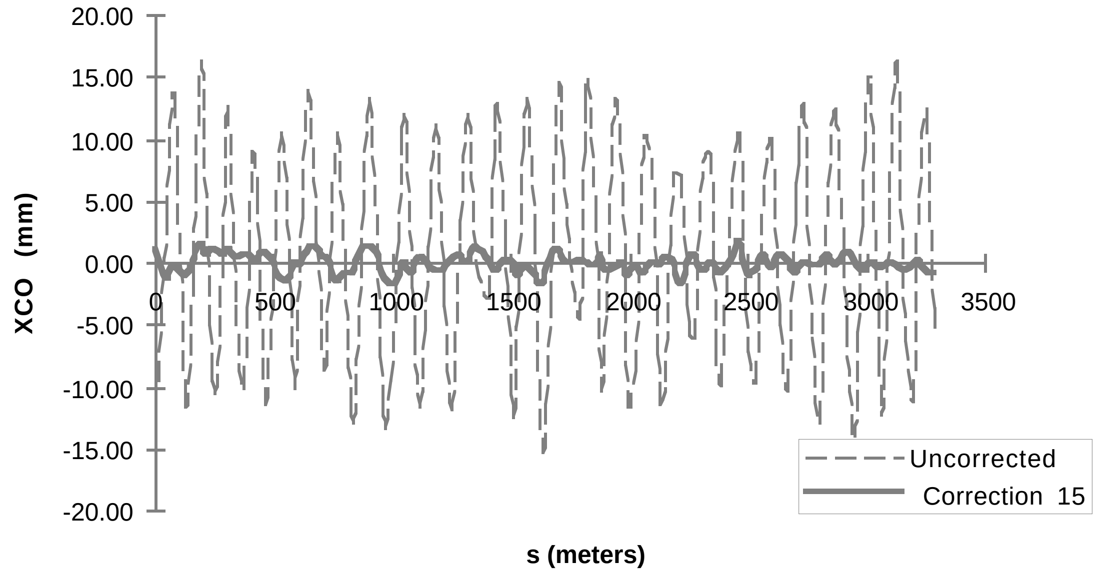

Figure 2.3.2: The uncorrected and corrected orbit corresponding to seed 8 in table 2.3.4. The corrected orbit is produced by transverse moves of fifteen magnets around the Recycler.

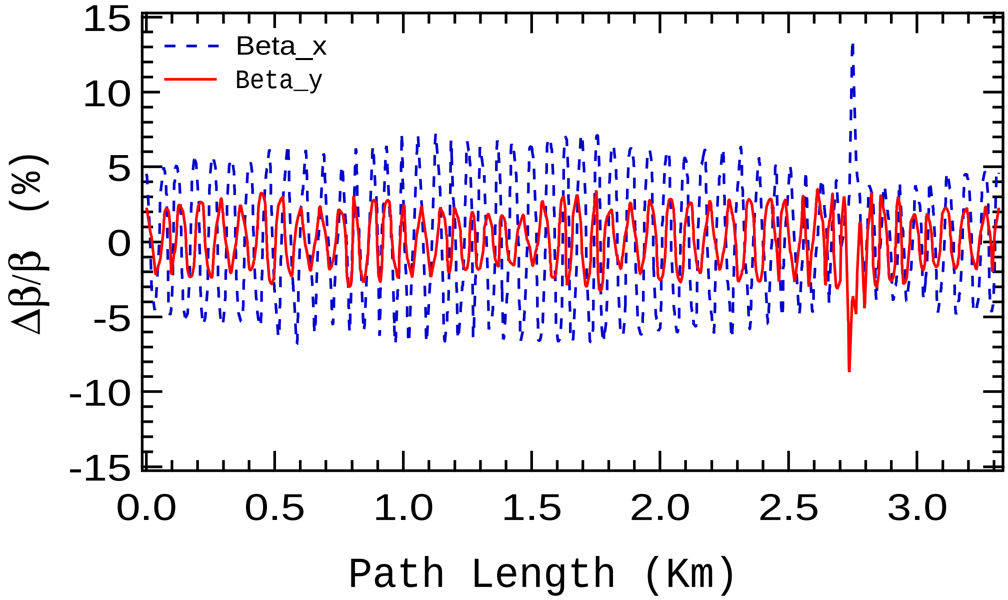

Figure 2.3.3: Typical beta function variation resulting from expected gradient strength errors.

2.3.4 Field Non-uniformities

Magnet field non-uniformities can be characterized in terms of field multipoles. This characterization is based on a description of the field in transverse coordinates via:

$$B_y + iB_x = B_0 \sum_{n=0}^{\infty} (b_n + ia_n)(x + iy)^n \quad , \quad (2.3.4)$$

where b_0, b_1, b_2 . . . are the normal dipole, quadrupole, sextupole, . . . field components, and a_0, a_1, a_2 . . . are the corresponding skew components. These higher-order field components can produce tune variations with particle oscillation amplitude and can produce unstable motion when the tune operating point comes close to fulfilling the relation

$$l\nu_x + m\nu_y = k \quad , \quad (2.3.5)$$

where l, m, and k are integers. The integers l and m are related to the field index n through the relation n=l+m-1.

The design tune of the Recycler is (Q_x,Q_y)=(25.425,24.415). This leaves it relatively isolated from resonances up to fifth order. Tune variations with amplitude and one dimensional resonance widths up to tenth order have been calculated for one Fermilab "unit" of multipole component, where Fermilab units are defined as the ratio of the multipole strength to the dipole strength, measured at 1", times 10,000, i.e.

$$1\text{unit} = b_n x^n \Big|_{x=1''} \times 10^4 = \frac{B^{(n)} x^n}{n! B_0}\Big|_{x=1''} \times 10^4 \quad . \quad (2.3.6)$$

In quadrupole magnets the multipole strengths are referred to the quadrupole, rather than the dipole field. Resonance widths generated by higher order multipoles are much less significant than tune variations with amplitude for the design tune point.

Sensitivity of the tune variation with amplitude to a given multipole contribution is computed using the expressions:

$$\Delta\nu_n = \frac{\varepsilon^{(n-1)/2}(n+1)!}{2^{n+2}\pi(\frac{n+1}{2})!^2} \sum_i b_{n,i}\theta_i\beta_i^{(n+1)/2}(39.37)^n \times 10^{-4} \quad , \quad (2.3.7)$$

$$\Delta\nu_n = \frac{\varepsilon^{(n-2)/2} n(n)!}{2^{n+1}\pi(\frac{n}{2})!^2} \frac{\Delta p}{p} \sum_i b_{n,i}\theta_i D_{x_i}\beta_i^{n/2}(39.37)^n \times 10^{-4} \quad . \quad (2.3.8)$$

Here all lengths are measured in meters, θ_i is the bend angle in the magnet, D_x is the horizontal dispersion function, and the particle oscillation amplitude is given by

$x(s) = A\sqrt{\beta(s)}\cos\phi(s) = \sqrt{\varepsilon\beta(s)}\cos\phi(s)$. Equation 2.3.7 applies only to n=odd multipoles and equation 2.3.8 to even multipoles. Equation 2.3.8 represents the feed down effect in the presence of non-zero dispersion and only leads to tune shifts for off-momentum particles.

Table 2.3.5 displays the tune shift for a particle with a 6.3 π mmmr (60π/βγ) oscillation amplitude, accompanied by a momentum offset of 0.3%, generated by a zeroth harmonic multipole contribution of one unit. This tune shift scales as $A^{(n-1)/2}$ where A is the amplitude and n is the field index in equation 2.3.4.

Table 2.3.5: Tune shift for a particle with an oscillation amplitude corresponding to 60 π mmmr and a momentum offset of 0.3% arising from one unit of the multipole component indicated in the left column. CF and Q refer to contributions from the combined function and quadrupole magnets respectively.

n	Multipole	Systematic		Random (rms)	
		$\Delta\nu$(CF)	$\Delta\nu$(Q)	$\Delta\nu$(CF)	$\Delta\nu$(Q)
3	Octupole	0.0170	0.0003	0.0017	0.0000
4	10-pole	0.0122	0.0002	0.0012	0.0000
5	12-pole	0.0074	0.0001	0.0007	0.0000
6	14-pole	0.0079	0.0001	0.0008	0.0000
7	16-pole	0.0033	0.0001	0.0003	0.0000
8	18-pole	0.0048	0.0001	0.0005	0.0000
9	20-pole	0.0015	0.0000	0.0002	0.0000

2.3.5 Tracking calculations

The tune vs. amplitude analysis presented above provides a guide as to the allowed multipole content of the magnets making up the Recycler. However, the Recycler is required to circulate beam for several hours and long term behavior in the presence of the full collection of misalignment and field errors is best understood through tracking simulations.

Performance in the presence of a mixture of alignment and magnetic field errors in the Recycler is tested by launching an array of particles at different amplitudes in the presence of the full array of errors described in sections 2.3.2-4. Particles are tracked with betatron oscillation amplitudes relative to a corrected closed orbit. Following introduction of errors the tune is adjusted to the nominal tune for a zero momentum offset particle using the phase trombone. The criterion for acceptable performance is survival over 10^5 turns (1 second of real time) with an oscillation amplitude corresponding to 60 π mmmr over the full momentum range of ±0.3%. The emittance of the recycled beam is expected to lie in the 20-25 π mmmr range, while 0.3% corresponds to the momentum acceptance of the stochastic cooling systems.

Specifically, particles are launched with a horizontal displacement "A" and a vertical displacement $\sqrt{\beta_y/\beta_x}\,A$, i.e. the particle has equal horizontal and vertical emittance. In translating into acceptance the displacement is referred to a horizontal beta function of 65 m ($x_{Launch} = \sqrt{\beta_x/65}\,A$). Particle are tracked with a constant momentum offset of,

$\Delta p/p$, which is varied over the range ±0.3%. Net chromaticities in both planes are set to -2, the correct sign to combat the bunched-beam head-tail instability. Particles with amplitudes varying from 15 mm to 35 mm are considered. Simulations are performed for five different seeds with a maximum of 100,000 turns tracked.

Combined function multipole component errors used in these simulations are given in table 2.3.6. As a starting point application of a criterion of detuning of less than 0.03 at 60 π mmmr was applied to table 2.3.5 in the presence of a 0.3% momentum offset. These multipoles were observed to result in a dynamic aperture somewhat less than the goal of 60π. As a result the multipole composition was varied with the set presented below leading to acceptable behavior. This set provides the basis of the current magnet specification as described in section 2.3.6. Further iteration on this set is anticipated. Multipole component errors used for the quadrupole magnets are given in table 2.3.7. Performance is relatively insensitive to these multipoles (at the factor of three level). Note that all systematic multipoles are modeled with the same sign. This is the most pessimistic assumption possible because it results in the maximum detuning with amplitude. It is worth noting that in the absence of any ≥8-pole errors the aperture of the Recycler is in excess of 100 π mmmr over the entire specified momentum range.

Table 2.3.6 Combined function magnet field multipoles used in the tracking simulations represented in figure 2.3.4. All multipoles are given in Fermilab "units" as referenced to the dipole component.

Multipole Component	Normal (Systematic)	Normal (Random)	Skew (Systematic)	Skew (Random)
Quadrupole	1.0	1.0	1.0	1.0
Sextupole	0.5	1.0	0	0.5
Octupole	0.5	0.5	0	0.5
10-pole	0.2	0.5	0	0.5
12-pole	0.1	0.5	0	0.5
14-pole	0.1	0.5	0	0.5
16-pole	0.1	0.5	0	0.5
18-pole	0.1	0.5	0	0.5
20-pole	0.1	0.5	0	0.5

Table 2.3.7: Quadrupole magnet field multipoles used in the tracking simulations represented in figure 2.3.4. All multipoles are given in Fermilab "units" as referenced to the quadrupole component.

Multipole Component	Normal (Systematic)	Normal (Random)	Skew (Systematic)	Skew (Random)
Quadrupole	0	8	0	0
Sextupole	0.5	0.5	0	0.5
Octupole	0.2	0.5	0	0.5
10-pole	0.1	0.5	0	0.5
12-pole	0.1	0.5	0	0.5
14-pole	0.1	0.5	0	0.5
16-pole	0.1	0.5	0	0.5
18-pole	0.1	0.5	0	0.5

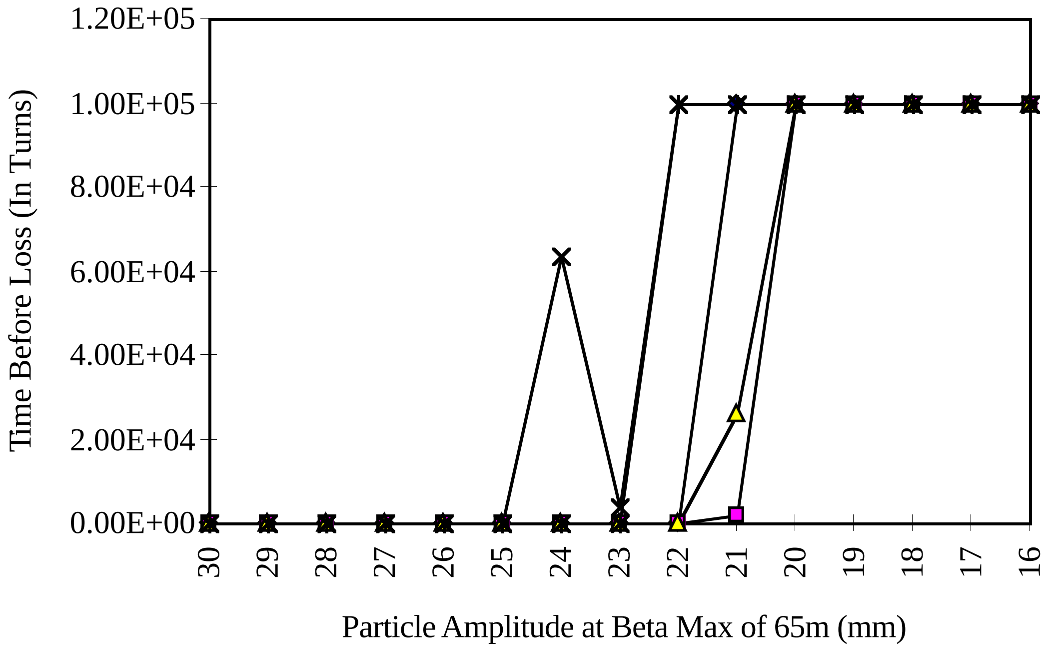

Figure 2.3.4: Survival plot for the Recycler. The number of turns survived, up to 100,000 turns, is shown as a function of the launch amplitude for five different collections of systematic and random alignment and magnetic field errors and a momentum offset of 0.3%.

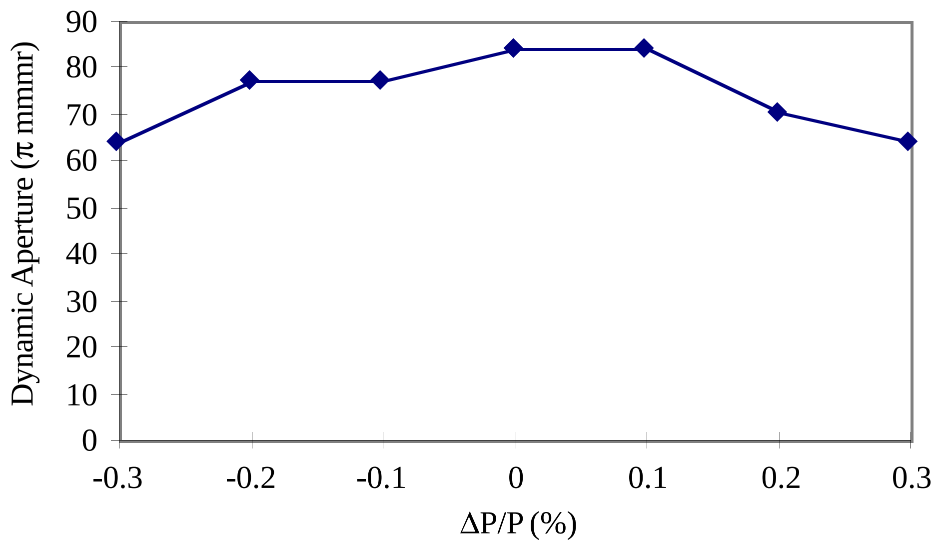

Figure 2.3.5: Dynamic aperture as a function of momentum offset over the full momentum range of the antiproton stack. This calculation is based on 100,000 turn survival and incorporates the full set of misalignment and field quality errors described in the text.

Figure 2.3.4 is a survival plot displaying how many turns a particle survives in the Recycler as a function of initial amplitude for each of five seeds. If the dynamic aperture of the machine is defined as the smallest amplitude particle that did not survive for 100,000 turns, then the dynamic aperture for the Recycler is predicted to be 22±1 mm, corresponding to a normalized emittance of 64±6 π mmmr.

Figure 2.3.5 shows the dynamic aperture as determined by 100,000 turn particle survival as a function of the momentum offset. This plot indicates that an aperture in excess of 60 π mmmr is maintained over the full momentum spread expected in the antiproton stack. Further tracking of up to 10^6 turns is currently being undertaken to understand longer term trends.

Particle tracking is also utilized to study the variation of the horizontal and vertical tunes as a function of oscillation amplitude. Figure 2.3.6 shows the tune space location as a function of amplitude for particles with a 0.3% momentum offset. Comparison with table 2.3.5 shows that the vertical tune shift with amplitude is nearly completely accounted for by the 0.5 unit systematic octupole component assumed in the tracking, while the other multipoles conspire to keep the horizontal tune shift small.

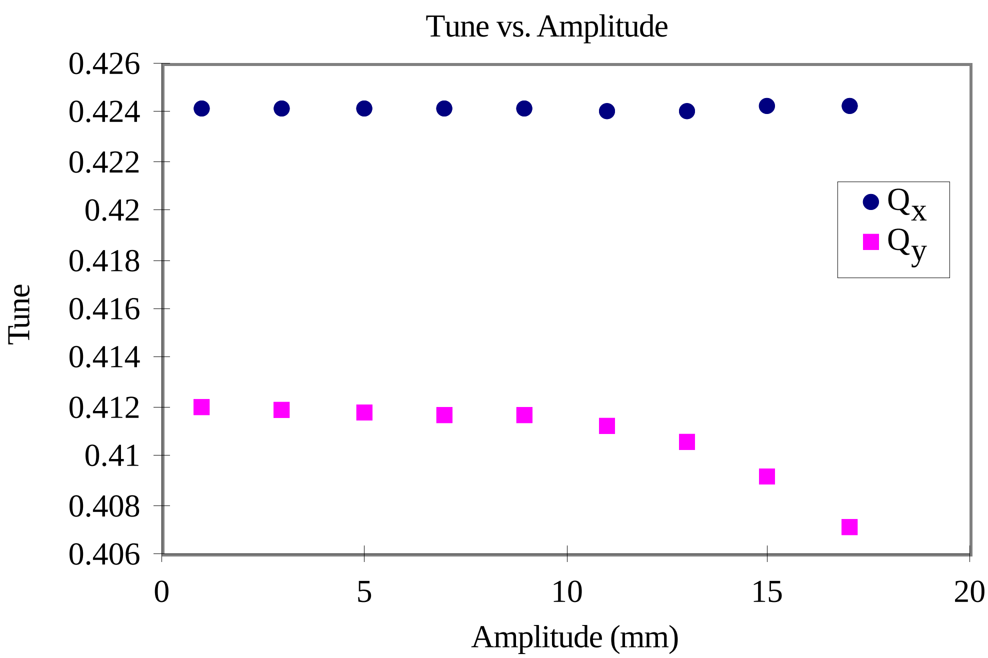

Figure: 2.3.6: Tune vs. amplitude for the set of multipoles given in Table 2.3.6 as determined through tracking.

2.3.6 Magnetic performance specification

Magnet performance specifications have been developed based on the analyses described in the above sections. The detailed performance specification is given in MI-0170 and is summarized here.

Tolerances have been specified for both systematic and random strength variations in the dipole, quadrupole, and sextupole field components, overall magnetic field uniformity across the required aperture, and allowed multipole composition. Criterion applied in deriving these tolerances are as follows:

1. Uncorrected closed orbit distortion due to magnetic field imperfections < 3 mm.

2. Beta function error due to magnetic field imperfections < 2%.

3. Tune relative to the nominal value within 0.1

4. Uncompensated minimum tune separation due to magnetic imperfections < .005

5. Tune variation with amplitude <.01 out to 20 mm.

6. Chromaticity within range of the sextupole correction system.

7. Beam survival at 20 mm oscillation amplitude for 5 seeds at 10^5 turns over the full ±0.3% range of momentum offsets.

Tolerances are summarized in tables 2.3.8 and 2.3.9 below.

Table 2.3.8: Magnet strength tolerances for Recycler combined function magnets.

Performance Measure	Tolerance (Systematic)	Tolerance (Random, rms)
Absolute bending strength of long combined function magnet	$5x10^{-4}$	$5x10^{-4}$
Ratio of short/long bending strength in combined function magnets	$5x10^{-4}$	$5x10^{-4}$
Ratio of gradient to nominal dipole in combined function magnets	$2x10^{-4}$/inch	$1x10^{-4}$/inch
Ratio of sextupole/dipole in long combined function magnets	$1x10^{-4}$/inch^2	$1x10^{-4}$/inch^2
Field flatness over ±20 mm	$\pm 1.5x10^{-4}$	$\pm 1.5x10^{-4}$

Table 2.3.9: Allowed multipole components, in Fermilab units, for combined function magnets. For quadrupole and sextupole components, the number listed is relative to the nominal design value.

Multipole Component	Normal (Systematic)	Normal (Random)	Skew (Systematic)	Skew (Random)
Quadrupole	1	1	1	1
Sextupole	0.5	1	-	0.5
Octupole	0.5	0.5	-	0.5
10-pole	0.2	0.5	-	0.5
12-pole	0.1	0.5	-	0.5
14-pole	0.1	0.5	-	0.5
16-pole	0.1	0.5	-	0.5
18-pole	0.1	0.5	-	0.5
20-pole	0.1	0.5	-	0.5

2.3.7 Required correction systems

Correction strategies and/or systems will be required to compensate for the magnetic field imperfections and misalignment effects described above. Systems or strategies are required for closed orbit correction, tune and chromaticity adjustment, coupling correction, and higher order resonance correction.

It is proposed that the closed orbit be corrected through adjustment of the transverse positions of a limited number of gradient magnets as described in section 2.3.2. No active correction elements are included in the design other than in the injection and extraction areas. As discussed in section 2.3.2 the uncorrected closed orbit is expected to show peak excursions of up to ~15 mm. A model in which fifteen gradient magnets are chosen to be moved was analyzed and shown to produce a corrected orbit distortion of <1 mm (rms) with peak distortions under 3 mm. The maximum transverse movement required is typically 2 mm, well within the range of tolerable mechanical motion allowed.

Tune adjustment is provided in the MI-60 phase trombone as described in section 2.2. Five families of two adjustable magnets each are provided. A total tuning range of ±0.5 is achievable with no perturbation of the optical functions outside the trombone region.

A chromaticity control (sextupole) magnet system will be incorporated into the Recycler. The incorporation of a design systematic sextupole component into the long combined function magnets will produce an uncorrected chromaticity in the Recycler of -2 in both transverse planes. A correction system is specified that allows adjustment of the chromaticity over a ±5 range relative to nominal in each plane. The total required strength of focusing and defocusing sextupoles as a function of chromaticity is given in table 2.3.10. As is shown a total strength (B"L) of 425 kG/m is required. Such a correction system will be implemented by utilizing unused Main Ring sextupoles. It is proposed to utilized two sets of sextupoles, 8 focusing and 16 defocusing, each to provide chromaticity control in the Recycler. Each set will be constituted of several pairs of magnets separated by 180° of phase in order to nullify any contribution to the third order resonant driving term.

Table 2.3.10: Sextupole strength required to achieve the Recycler chromaticities listed in the left hand columns. B"L is the integrated strength and $N_{F,D}$ is the number of magnets in each family. It is proposed to utilize a system in which N_F=8 and N_D=16.

$\Delta\xi_H$	$\Delta\xi_V$	$N_F(B''L)_F$ (kG/m)	$N_D(B''L)_D$ (kG/m)
+5	0	176	-69
0	+5	34	-355
+5	+5	211	-424

As described in section 2.3.1 coupling due to systematic and random skew quadrupole components, and due to alignment errors could be significant in the Recycler. Global coupling will be controlled via two skew quadrupole circuits. The location of these magnets depends on details of the lattice and will not be defined until the lattice is frozen. All tracking simulations performed to date incorporate uncompensated coupling.

2.3.8 Summary

Calculations presented in this section have shown that the Recycler ring can achieve the design specification for storage of beam with a 40 π mmmr admittance and a momentum spread of 0.3%. This performance requires magnets built to tolerances listed in section 2.3.6, alignment at the level described in section 2.3.2, and correction of the closed orbit. Magnet requirements are dictated by the allowable orbit and beta function distortions, and by the dynamic aperture in the presence of higher order magnetic multipoles.

It is anticipated that once full length prototype Recycler magnets have been measured the studies described here will be expanded more systematically to construct a performance model of the real Recycler ring.