Energy Distribution in a Relativistic DC Electron Beam

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Electron cooling of the 8.9 GeV/c antiprotons in the Recycler ring requires high-quality dc electron beam with the current of several hundred mA and the kinetic energy of 4.3 MeV. The only technically feasible way to attain such high electron currents is through beam recirculation (charge recovery). The primary current path is from the cathode at high voltage terminal to ground where the electron beam interacts with the antiproton beam and cooling takes place, then to the collector located in the terminal, and finally through the collector power supply back to the cathode. The energy distribution function of the electron beam after the deceleration determines the required collector energy acceptance. Multiple and single intra-beam scattering as well as the dissipation of density micro-fluctuations during the beam transport are studied as factors forming a core and tails of the electron energy distribution. For parameters of the Fermilab electron cooling project[1], the single intra-beam scattering (Touschek effect) is found to be of the most importance.

Introduction

The electron cooling device, which is under construction at Fermilab[1], employs an electrostatic acceleration and deceleration to generate a dc electron beam. Some parameters of this device are listed in Table 1.

Table 1. Parameters of the electron cooling device considered in this paper.

Parameter	Symbol	Value	Units
Beam current	I_e	1	A
Electron momentum	p	4.8	MeV/c
Cathode radius	a_c	0.25	cm
Electron temperature at the cathode	T_c	0.1	eV
Length of electron trajectory	l	80	m
Collector potential with respect to the cathode	U_{coll}	2 - 4	kV

A scheme with the electron beam recirculation allows to use a 4.3-MW power beam for cooling, while the energy dissipated in the electron beam collector is as low as 2-4 kW. On the other hand, a stable operation of such a device is possible only if the current loss is very low. In the initial recirculation experiment[2], multi-hours operation was kept uninterrupted only when the lost current δI_e was below 10 μA. One of the possible reasons for the loss is the electron energy spread. Obviously, if the energy of some electrons is decreased by more than eU_{coll} during the transport, these electrons are repelled from the collector and lost. Hence, the slowest tail of the energy distribution, containing $\Delta \equiv \delta I_e / I_e \cong 10^{-5} - 10^{-6}$ portion of the beam, can affect the collector design and the U_{coll} value. The energy distribution function is formed primarily by the intra-beam scattering (IBS) and by the dissipation of density micro-fluctuations during the beam transport. An electron with a longitudinal velocity $v_{\|} << c$ in the beam frame has the following energy deviation in the laboratory frame:

$$U = pv_{\|} = \gamma\beta mcv_{\|} . \tag{1}$$

Beam acceleration and deceleration preserves this value. A consequence of this formula is that kinematically the beam longitudinal temperature decreases with the beam energy,

$$T_{\|} \equiv m\overline{v_{\|}^{\,2}} \cong mT_c^2 / p^2$$

where T_c is the initial (cathode) beam temperature. Normally, this value is so small that the actual longitudinal temperature is determined by other factors.

Core of the Distribution

Two phenomena determine the r.m.s. energy spread in the electron beam: the multiple IBS and the dissipation of density micro-fluctuations.

a. Multiple Intra-Beam Scattering

Normally, the longitudinal temperature of the accelerated beam is much smaller than the transverse, or $v_{\|} << v_{\perp} \equiv \sqrt{\overline{\left(\mathbf{v} - \overline{\mathbf{v}}(\mathbf{r})\right)^2}}$ in terms of the velocities in the beam frame. In this case, the Vlasov equation with the Landau collision integral[3] reduces to a one-dimensional diffusion equation:

$$\frac{\partial f_{\|}}{\partial t} = \frac{D}{2} \frac{\partial^2 f_{\|}}{\partial v_{\|}^{\,2}} , \tag{2}$$

where the diffusion coefficient,

$$D = 4\pi n_e r_e^2 c^4 L_C \iint d^2\mathbf{v} d^2\mathbf{v}' \frac{f_{\perp}(\mathbf{v}) f_{\perp}(\mathbf{v}')}{|\,\mathbf{v} - \mathbf{v}'|} ,$$

is independent of the longitudinal velocity. Here n_e is the electron density (constant over a beam cross-section), $\mathbf{v}$ is a 2D vector of the transverse velocity, and

$$L_C = \ln\left(\frac{r_D}{r_{\min}}\right); \quad r_{\min} = \frac{r_e c^2}{v_\perp^2}$$

is the Coulomb logarithm with $r_D = v_\perp / \omega_e$ as the Debye radius , all the values are taken in the beam frame. For a Gaussian distribution

$$f_\perp(\mathbf{v}) = \frac{\exp\left(-\mathbf{v}^2 / 2v_\perp^2\right)}{2\pi v_\perp^2}$$

it yields

$$D = 2\pi^{3/2} n_e r_e^2 c^4 L_C / v_\perp \ .$$

This result agrees with the corresponding formula in Ref.[4] derived on the basis of Eqs. (76, 77) of Ref.[5] and is two time larger than what was reported in Ref[6].

In a more general case of a Gaussian distribution with arbitrary (unequal) transverse r.m.s. velocities $\overline{v_x^2}$, $\overline{v_y^2}$ and a constant electron beam density over the cross-section, it results in

$$D = \frac{2\pi^{3/2} n_e r_e^2 c^4 L_C}{\sqrt{\overline{v_x^2}\ \overline{v_y^2}}} \int_0^{2\pi} \frac{d\varphi}{2\pi} \frac{1}{\sqrt{\dfrac{\cos^2\varphi}{\overline{v_x^2}} + \dfrac{\sin^2\varphi}{\overline{v_y^2}}}} \, .$$

The diffusion equation (2), solved with the delta-function initial condition, $f_\parallel(v_\parallel, 0) = \delta(v_\parallel)$, leads to the Gaussian distribution

$$f_\parallel(v_\parallel, t) = \frac{\exp\left(-v_\parallel^2 / 2\overline{v_\parallel^2}(t)\right)}{\sqrt{2\pi \overline{v_\parallel^2}(t)}} \, , \quad \overline{v_\parallel^2}(t) = Dt \tag{3}$$

Taking into account the relation between the velocity in the beam frame and the energy offset in the laboratory frame (1), the lab frame r.m.s. energy spread follows as an integral over the beam line:

$$\overline{U_{IBS}^2} = \frac{2\sqrt{\pi} I_e r_e^2 L_C}{e c^2 \varepsilon_{4n}} \left(mc^2\right)^2 \int_0^l dz \int_0^{2\pi} \frac{d\varphi}{2\pi} \frac{1}{\sqrt{\dfrac{\cos^2\varphi}{\overline{v_x^2}} + \dfrac{\sin^2\varphi}{\overline{v_y^2}}}} \tag{4}$$

where I_e is the electron current, $\varepsilon_{4n} = \left(a_x^2 a_y^2 \overline{v_x^2}\ \overline{v_y^2}\right)^{1/2} / c^2$ is the normalized 4D phase space emittance with a_x, a_y as half-axes of the beam with an elliptical cross-section, and *l* is a length of the transport in the lab frame. The normalized emittance is invariant along the transport line; it is determined by the cathode radius a_c and its temperature T_c as $\varepsilon_{4n} = a_c{}^2 T_c / mc^2$; the r.m.s. velocities are expressed as $\overline{v_{x,y}^2} = \varepsilon_{4n} c^2 / a_{x,y}^2$. Note that the energy spread does not depend on the beam energy; therefore, the formula (4) can be directly applied to the regions where the beam is accelerated or decelerated. For parameters of the Fermilab electron cooling project with $L_C = 8$, and $a_x \approx a_y \approx 0.6\,\text{cm}$ it gives $\sqrt{\overline{U_{IBS}^2}} = 110\,\text{eV}$.

b. Dissipation of Density Fluctuations

One more factor of the energy spread widening is the dissipation of the density micro-fluctuations in the electron beam after acceleration. An excessive potential energy $\cong Ce^2 n_e{}^{1/3}$ (beam frame) is transformed into the longitudinal temperature, giving rise to the r.m.s. energy spread

$$\overline{U_{DF}^2} \cong C(\gamma\beta)^{5/3} r_e \left(\frac{I_e}{\pi eca_e^2}\right)^{1/3} \left(mc^2\right)^2$$

Here *C* is a numerical coefficient of about 1 estimated in Ref[6] as *C*=1.9. For the mentioned parameters, it yields $\sqrt{\overline{U_{DF}^2}} \cong 80\,\text{eV}$. The actual contribution of the effect can be somewhat less because the electrons make only about one plasma oscillation during their drift at the maximum energy.

The contribution of the micro-fluctuations slightly changes the spread resulting from the IBS: $\sqrt{\overline{U^2}} = \sqrt{\overline{U_{IBS}^2} + \overline{U_{DF}^2}} = 130\,\text{eV}$. Note that this spread is not significant for the electron cooling process, which in any scenario could tolerate an order of magnitude higher energy spread.

Tails of the Distribution

a. Gaussian Tails of the Core

The Gaussian distribution (3) can be also presented as

$$f_{\|}(U) = \frac{\exp\left(-U^2 / 2\overline{U^2}\right)}{\sqrt{2\pi\overline{U^2}}} \ .$$

A portion of particles $\Delta(U)$ with the energy deviation exceeding a given value U will be referred to as losses. For the Gaussian distribution the losses are given by the complementary error function $\mathrm{erfc}(x) \equiv (2/\sqrt{\pi})\int_x^\infty \exp(-x^2)dx \cong \exp(-x^2)/(x\sqrt{\pi})$ as

$$\Delta(U) = (1/2)\,\mathrm{erfc}\left(U/\sqrt{2\overline{U^2}}\right) . \tag{5}$$

It follows from Eq. (5) that the energy threshold corresponding to the level of losses of $\Delta(U) = 1\cdot 10^{-6}$ is $U = 4.75\sqrt{\overline{U^2}}$. For the parameters discussed here it gives $U = 580\,\mathrm{eV}$. In the following subsection it is shown though that the losses, caused by the Touschek effect (single IBS), significantly exceed this Gaussian tail contribution when the losses level is low enough.

b. Touschek Effect

When relative velocities of the scattered electrons are smaller then $\alpha c \approx c/137$, the classical Rutherford formula for the differential cross-section can be used. In a symmetrized form for both particles, it looks as

$$\frac{d\sigma}{do} = \frac{r_e^2 c^4}{v_0^4}\left(\frac{1}{\sin^4(\chi/2)} + \frac{1}{\cos^4(\chi/2)}\right),$$

where v_0 is the relative velocity of the electrons and χ is the scattering angle in the center-of-mass system. The losses calculation is made below following the methods of Refs.[7],[8]. The scattering event can be described by two angles: the angle θ between the final relative velocity and the longitudinal axis and the angle φ between a projection of the final relative velocity on the transverse plane and the initial relative velocity. The introduced angles are related to each other as $\cos^2\chi = \sin^2\theta\cos^2\varphi$. The cross-section for scattering events in which a particle acquires the longitudinal velocity larger than $v_\parallel$ is equal to

$$\sigma(v_0, v_\parallel) = \begin{cases} \displaystyle\int \frac{d\sigma}{do}do = \int_0^{2\pi} d\varphi \cdot 2 \int_0^{\arccos(2v_\parallel/v_0)} \frac{d\sigma}{do}\sin\theta\, d\theta = \frac{4\pi r_e^2 c^4}{v_0^2 v_\parallel^2}\left(1 - \frac{4v_\parallel^2}{v_0^2}\right), & \text{if } 4v_\parallel^2 < v_0^2 \\ 0, \quad \text{otherwise} \end{cases} .$$

The instantaneous loss rate follows as

$$\frac{1}{n_e}\frac{dn_e}{dt} = \frac{n_e}{2}\int d\mathbf{v}d\mathbf{v}'\sigma(|\mathbf{v}-\mathbf{v}'|, v_\parallel)\,|\mathbf{v}-\mathbf{v}'|\; f_\perp(\mathbf{v}) f_\perp(\mathbf{v}')$$

(beam frame); the factor ½ appears because the events are double-counted by the above integral. For the Gaussian distribution

$$f_{\perp}(v_x, v_y) = \frac{\exp\left(-v_x^{\,2} / 2\overline{v_x^2} - v_y^{\,2} / 2\overline{v_y^2}\right)}{2\pi\sqrt{\overline{v_x^2}\,\overline{v_y^2}}}$$

the above 4D integral reduces to a single integral[8] which allows for the losses to be presented as an integral along the beam line:

$$\Delta(U) = \frac{\sqrt{\pi} I_e r_e^2 L_C}{e c^2 \varepsilon_{4n}} \left(\frac{mc^2}{U}\right)^2 \int_0^l dz \int_0^{2\pi} \frac{d\varphi}{2\pi} \frac{\mathrm{S}(\Xi(\varphi))}{\sqrt{\dfrac{\cos^2\varphi}{\overline{v_x^2}} + \dfrac{\sin^2\varphi}{\overline{v_y^2}}}} \quad . \tag{6}$$

Here

$$\Xi(\phi) = \frac{U}{p}\sqrt{\frac{\cos^2\phi}{\overline{v_x^2}} + \frac{\sin^2\phi}{\overline{v_y^2}}}$$

is basically a ratio of the longitudinal velocity in the beam frame to its thermal transverse velocity, and

$$\mathrm{S}(x) \equiv \frac{\exp(-x^2)}{\sqrt{\pi}} \int_0^{\infty} \frac{y\exp(-y)}{\left(y + x^2\right)^{3/2}} dy = \left(1 + 2x^2\right)\mathrm{erfc}(x) - \frac{2x}{\sqrt{\pi}}\exp(-x^2) \cong \frac{\exp(-x^2)}{\sqrt{\pi}x^3}\left(1 - \frac{3}{x^2} + O\left(x^{-4}\right)\right)$$

is a special function, which is convenient to introduce. Until the longitudinal velocity reaches the value of both transverse thermal velocities,

$$v_{\parallel}^2 \equiv U^2 / p^2 < \max\left(\overline{v_x^2}\ , \overline{v_y^2}\right),$$

the Touschek losses (6) do not contain any exponential factor, decreasing with energy as slowly as its second power $\Delta(U) \propto 1/U^2$. It means that the tails of the energy distribution due to single IBS are orders of magnitude higher than the ones resulting from the multiple IBS (5). When the longitudinal velocity is smaller than both transverse thermal velocities, the Touschek losses (6) are simply related to the core spread caused by the multiple IBS (4) as:

$$\Delta(U) = \frac{\overline{U_{IBS}^2}}{2L_C U^2} \quad , \quad \text{if} \quad v_{\parallel}^2 \equiv U^2 / p^2 << \min\left(\overline{v_x^2}\ , \overline{v_y^2}\right).$$

In the opposite limiting case of $v_{\parallel}^2 >> \overline{v_{\perp}^2}$, the asymptotic calculations of the loss integral can be performed. For a round beam with $\overline{v_x^2} = \overline{v_y^2} \equiv v_{\perp}^2$ it yields

$$\Delta(U) = \frac{I_e r_e^2 L_C}{ec^2 \varepsilon_n^2}\left(\frac{mc^2}{U}\right)^2 \int_0^l dz\, v_\perp \left(\frac{pv_\perp}{U}\right)^3 \exp\left(-\frac{U^2}{p^2 v_\perp^2}\right) .$$

In this asymptotic case, the losses decrease very rapidly with the limiting energy, U. When the envelope oscillations along the beam line are insignificant, the inverse problem of finding the required energy acceptance U for given losses is weakly sensitive to the value of these losses, giving practically in any case $U \cong pv_\perp$. For a 5-mm radius round beam and the beam parameters listed in Table 1, $pv_\perp \approx$ 1.5 keV. When the envelope variations are significant, parts of the beam trajectory with the highest temperature (the smallest beam size) mainly determine the loss integral (6).

For the Fermilab electron cooling project, the designed beam envelope is presented in Fig.1.

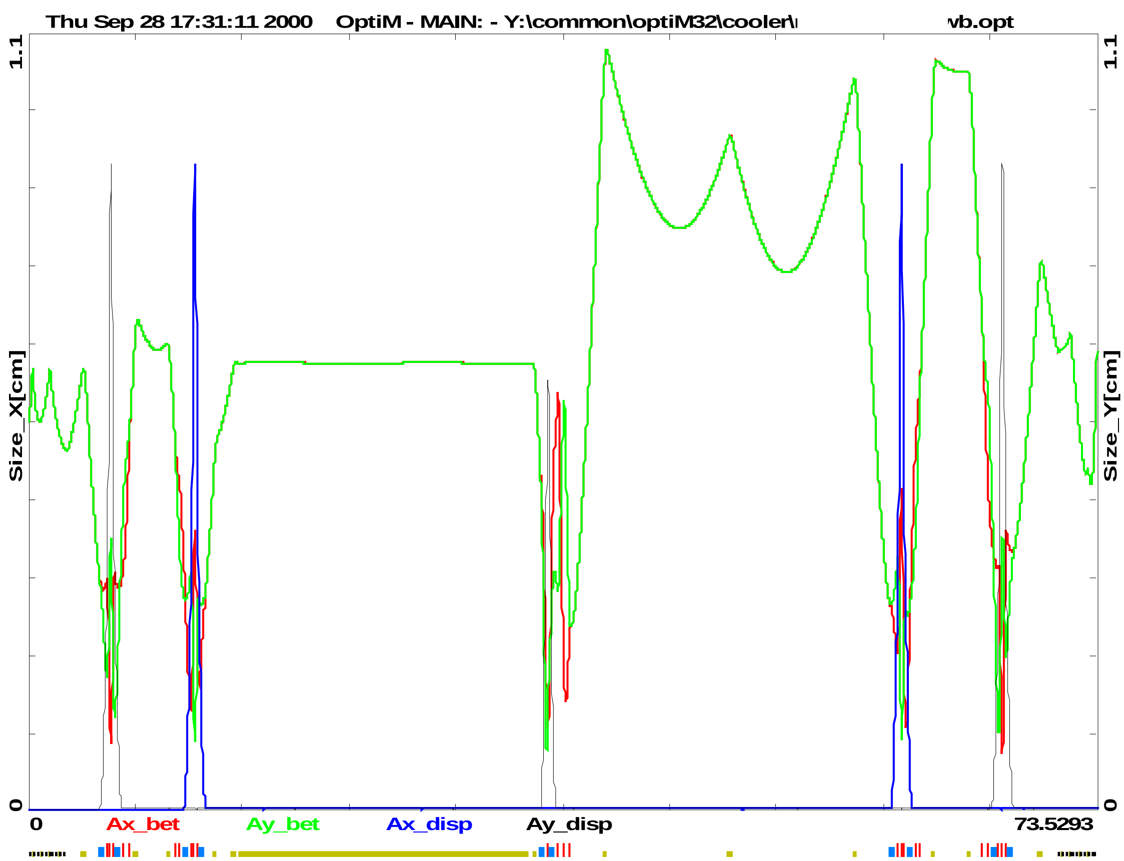

Figure 1: Beam envelopes (green and red lines) and dispersion functions (black and blue) in the designed electron transport line for the Fermilab electron cooling system.

For this envelope, the Touschek losses at the interesting level $\cong 10^{-5} - 10^{-6}$ are mainly determined by the bend portions of the trajectory where the beam is the smallest, and, consequently, the hottest. Note that the beam is also non-round there. For this design, the losses (6) were integrated numerically; the result is presented in Fig.2.

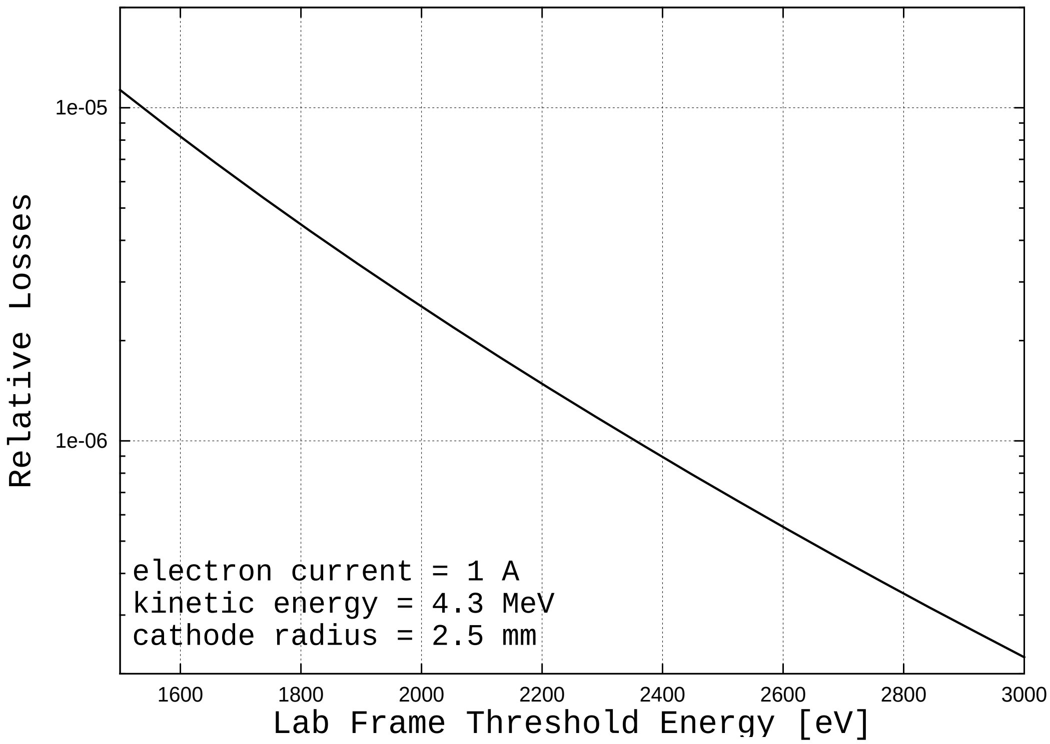

Figure 2: The Touschek losses as a function of the lab-frame threshold energy *U*, numerically calculated using the beam envelope in Fig. 1.

It is interesting to note that the beam magnetization at the cathode increases the average beam size in the transport line and, thus, reduces the acquired energy spread. For an angular momentum dominated beam[9], minimal beam size inside the bends increases with the magnetic flux Φ at the cathode as $\propto \sqrt{\Phi}$. Therefore, the maximum transverse temperature goes as $\propto 1/\Phi$. Thus, the flux increase diminishes the transverse temperature present in the exponent of the Touschek losses. The r.m.s. energy spread also goes down with the flux as $\propto 1/\sqrt{\Phi}$.

Other phenomena, such as scattering on the cooled antiprotons, elastic and inelastic scattering on the residual gas were found to be insignificant for the discussed parameters.

Conclusions

1. Our numerical simulations show that the single IBS places a limitation for the minimum collector voltage, U_{coll}. To provide the acceptable level of losses of $\Delta \approx 2.5 \cdot 10^{-6}$, the value of U_{coll} should be above 2 kV. Detailed simulations of the

low-energy electron trajectories in the collector regions shall be done to take into account the beam space charge.

2. The rms energy spread of the beam due to the multiple IBS is below the acceptable limit for the electron cooling process.

3. Tails of the multiple IBS Gaussian distribution are insignificant in comparison with the single large-angle IBS tails.

4. Any beam size increase (due to the angular momentum or the beam space charge) in the beam line regions with fixed focusing properties (such as bends) is beneficial for both the multiple and single IBS as it leads to a lower transverse temperature and lower beam density.

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