Chapter 11

TRANSVERSE COUPLED BUNCH INSTABILITIES

11.1 Resistive Wall

If there are M identical equally spaced bunches in the ring, there are $\mu = 0, \cdots, M-1$ transverse coupled modes when the center-of-mass of one bunch leads its predecessor by the betatron phase of $2\pi\mu/M$. The transverse growth rate for the μ-th coupled-bunch mode is given by Eq. (10.45). Including chromaticity, it becomes

$$\frac{1}{\tau_{m\mu}} = -\frac{1}{1+m}\frac{eMI_bc}{4\pi\nu_\beta E_0}\frac{\sum_q \mathcal{R}e\, Z_1^\perp(\omega_q)h_m(\omega_q-\chi/\tau_L)}{B\sum_q h_m(\omega_q-\chi/\tau_L)}\ , \tag{11.1}$$

where $\omega_q = (qM+\mu)\omega_0 + \omega_\beta + m\omega_s$, the bunching factor $B = M\tau_L/T_0$ has been used, $\chi = \omega_\xi\tau_L$ is the chromaticity phase shift across the bunch of full length τ_L and T_0 is the revolution period. Here, we assume that all the bunches are executing synchrotron oscillations in the same longitudinal azimuthal mode m.

The most serious transverse coupled-bunch instability that occurs in nearly all storage rings is the one driven by the resistive wall [1]. Since* $\mathcal{R}e\, Z_1^\perp \propto \omega^{-1/2}$ and is positive (negative) when the angular frequency ω is positive (negative), the betatron line at the

*Here, we assume that the wall is thicker than one skin depth at revolution frequency. Otherwise, $\mathcal{R}e\, Z_1^\perp \propto \omega^{-1}$.

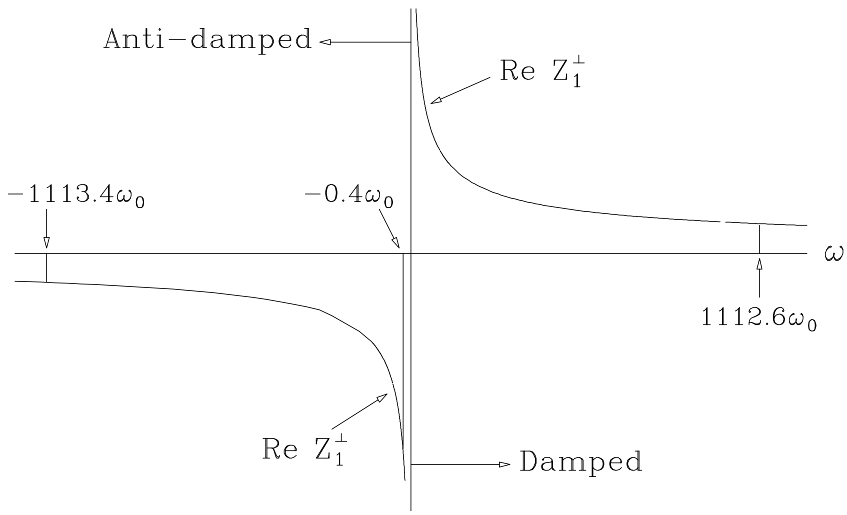

Figure 11.1: The $-0.4\omega_0$ betatron line in the Tevatron dominates over all other betatron lines for the $\mu = 1093$ mode coupled-bunch instability driven by the resistive wall impedance.

lowest negative frequency acts like a narrow resonance and drives transverse coupled-bunch instabilities. Take, for example, the Fermilab Tevatron in the fixed-target mode, where there are $M = 1113$ equally spaced bunches. The betatron tune is $\nu_\beta = 19.6$. The lowest-negative-betatron-frequency line is at $(qM+\mu)\omega_0 + \omega_\beta = -0.4\omega_0$, for mode $\mu = 1093$ and $q = -1$. The closest damped betatron line ($q = 0$) is at $(1113-0.4)\omega_0$, but $\mathcal{R}e\, Z_1^\perp$ is only $-\sqrt{0.4/1112.6}$ the value at $-0.4\omega_0$. The next anti-damped betatron line ($q = -2$) is at $-1113.4\omega_0$, with $\mathcal{R}e\, Z_1^\perp$ equal to $\sqrt{0.4/1113.4}$ the value at $-0.4\omega_0$. This is illustrated in Fig. 11.1. Thus, it is the $-0.4\omega_0$ betatron line that dominates. From Eq. (11.1), the growth rate for this mode can therefore be simplified to

$$\frac{1}{\tau_{m\mu}} \approx -\frac{1}{1+m}\frac{eMI_bc}{4\pi\nu_\beta E_0}\,\mathcal{R}e\, Z_1^\perp(\omega_q)F_m'(\omega_q\tau_L - \chi)\ , \tag{11.2}$$

where $\chi = \omega_\xi\tau_L$ and the form factor is

$$F_m'(\omega\tau_L) = \frac{2\pi h_m(\omega)}{\tau_L\displaystyle\int_{-\infty}^{\infty} h_m(\omega)d\omega}\ , \tag{11.3}$$

which is plotted in Fig. 11.2 for the sinusoidal modes. For zero chromaticity, only the $m = 0$ mode can be unstable because the power spectra for all the $m \neq 0$ modes are nearly zero near zero frequency. Since the perturbing betatron line is at extremely low

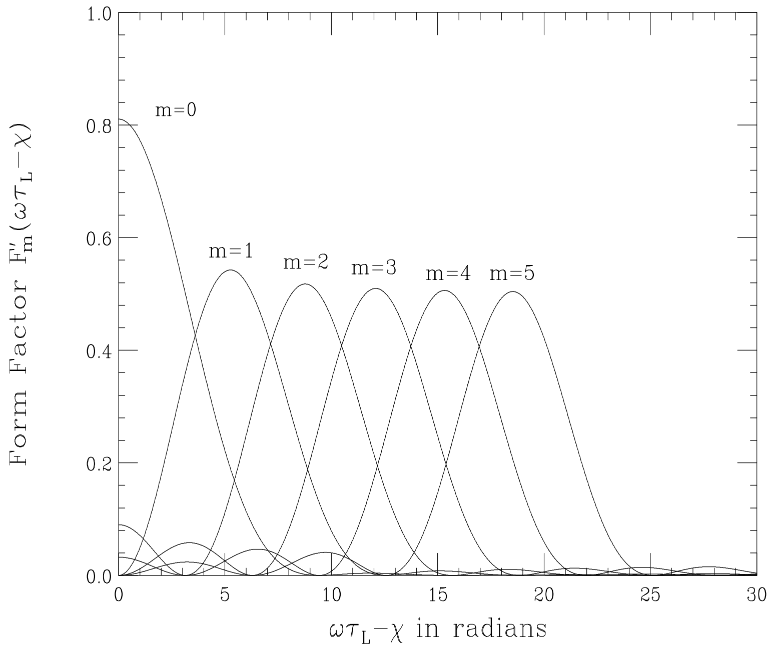

Figure 11.2: Plot of form factor $F'_m(\omega\tau_L - \chi)$ for modes $m = 0$ to 5. With the normalization in Eq. (10.43), these are exactly the power spectra h_m.

frequency, we can evaluate the form factor at zero frequency. For the sinusoidal modes, we get $F'_0(0) = 8/\pi^2 = 0.811$.

One method to make this coupled-bunch mode less unstable or even stable is by introducing positive chromaticity when the machine is above transition. For the Tevatron with slip factor $\eta = 0.0028$, total bunch length $\tau_L = 5$ ns, and revolution frequency $f_0 = 47.7$ kHz, a chromaticity of $\xi = +10$ will shift the spectra by the amount $\chi = \omega_\xi\tau_L = 2\pi f_0\xi\tau_L/\eta = 5.4$. The form factor and thus the growth rate is reduced by more than 4 times. However, from Figs. 7.5 and 10.5, we see that the spectra are shifted by $\omega_\xi\tau_L/\pi = 1.7$ and the $m = 1$ mode becomes unstable. Another method for damping the instability is to introduce a betatron angular frequency spread using octupoles, with the spread larger than the growth rate.

A third method is to employ a damper. Let us derive the displacements of consecutive bunches at a beam-position monitor (BPM). Suppose the first bunch is at the BPM with betatron phase $\phi_{\beta 0} = 0$; its displacement registered at the BPM is proportional to $\cos\phi_{\beta 0} = 1$. At that moment, the next bunch has phase $2\pi\bar{\mu}/M$ in advance, where $\bar{\mu} = qM + \mu = -20$. When this bunch arrives at the BPM, the time elapsed is T_0/M and

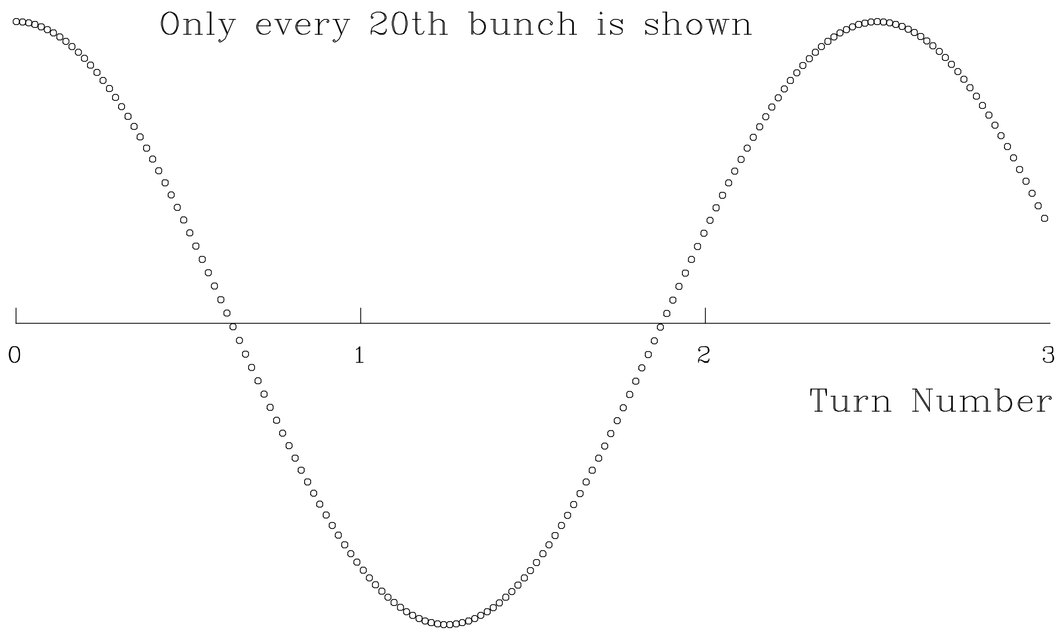

Figure 11.3: Difference signal at a BPM displaying the displacement of every 20th bunch, when the $\mu = 1093$ mode of transverse coupled-bunch is excited by the resistive wall impedance.

the change in betatron phase is $\omega_\beta T_0/M = 2\pi\nu_\beta/M$. The total betatron phase on arrival at the BPM is therefore $\phi_{\beta 1} = 2\pi\bar{\mu}/M + 2\pi\nu_\beta/M = 2\pi(\bar{\mu}+\nu_\beta)/M = (-0.4)2\pi/M$, and the displacement registered is $\cos\phi_{\beta 1}$ When the nth consecutive bunch arrives at the BPM, its phase will be $\phi_{\beta n} = n(-0.4)2\pi/M$. This is illustrated in Fig. 11.3 when the BPM is registering every 20th bunch [2]. What we see at the BPM is a wave of frequency -0.4 harmonic or about 19.1 kHz. Because we know that the bunches follow the pattern of such a slow wave, we only require a very narrow-band feedback system to damp the instability. Usually the adjacent modes $\mu = 1092,\ 1091,\ \cdots$ will also be unstable at the $-1.4\omega_0$, $-2.4\omega_0$, $\cdots$ betatron lines; but the growth rates will be smaller.

When all the $h = 1113$ rf buckets are filled with 6×10^{10} protons each in one scenario of the Tevatron in the fixed-target mode, the average total current is $MI_b = 0.511$ A. The vertical resistive-wall impedance has a real part $\mathcal{R}e\, Z_1^\perp = 43.74$ MΩ/m at the revolution harmonic. Thus, at $-0.4\omega_0$, it becomes $\mathcal{R}e\, Z_1^\perp = -69.16$ MΩ/m. At the injection energy of $E_0 = 150$ GeV and zero chromaticity, the transverse coupled-bunch growth rate driven by the resistive-wall impedance is $\tau_\mu^{-1} = 232$ s^{-1} and the growth time is 4.30 ms or 204 revolution turns. The mean radius of the Tevatron ring is $R = 1$ km. In fact, this growth time is more or less the same for all accelerator rings [3]. For example, preceding the Tevatron, there are the Main Injector and the Booster. All of them have the same 53-MHz rf. The Main Injector has 588 rf buckets and the Booster has 84 rf buckets. First, if all the buckets of each ring are filled, the average total current MI_b should be the same for all the 3 rings. Second, the beam energy E_0 scales as the size of the ring or the mean radius R and betatron tune ν_β scales as $\sqrt{R}$. Third, the resistive-wall impedance, as given by

$$Z_1^\perp(\omega) = [1 - i\,\mathrm{sgn}(\omega)]\frac{2Rc\rho}{\omega b^3 \delta_{\rm skin}} \tag{11.4}$$

in Eq. (1.44), where b is the beam pipe radius, $\delta_{\rm skin}$ is the skin depth, and ρ is the resistivity, scales as $R^{3/2}$ because the revolution frequency scales as R^{-1}. Substituting into Eq. (11.2), we find that the growth rate turns out to be independent of the size of the ring. Of course, usually there are differences in the vacuum chamber, and number of particles per bunch, and also the residual betatron tune. However, it is safe to say that the growth time of transverse couple-bunch instability for every completely filled accelerator ring should be of the order of a few to a few tens of milliseconds. Although the growth time is independent of the size of the ring, the growth time in turn number is inversely proportional to the size of the ring. Thus, for the Very Large Hadron Collider (VLHC) under consider consideration with a circumference of 233 km, the growth time will be only 5.5 revolution turns according to this scaling and assuming the residual tune to be $\frac{1}{2}$. For this reason, large machines will require powerful feedback systems, for example, criss-crossing feedback and/or one-turn correction scheme.

11.2 Narrow Resonances

The narrow higher-order transverse resonant modes of the rf cavities will also drive transverse coupled-bunch instabilities. The growth rates are described by the general growth formula of Eq. (11.1). When the resonance is narrow enough, only the betatron lines closest to the resonant frequency $\omega_r/(2\pi)$ contribute in the summation. The growth rate is therefore given by Eq. (11.2), where two betatron lines are included.

$$\frac{1}{\tau_{m\mu}} \approx -\frac{1}{1+m}\frac{eMI_bc}{4\pi\nu_\beta E_0}[\mathcal{R}e\, Z_1^\perp(\omega_q)F_m'(\omega_q\tau_L - \chi) - \mathcal{R}e\, Z_1^\perp(\omega_{q'})F_m'(\omega_{q'}\tau_L - \chi)]\ , \tag{11.5}$$

where q and q' satisfy

$$\begin{cases} -\omega_r \approx \omega_q = (qM + \mu + \nu_\beta + m\nu_s)\omega_0 \\ \;\;\omega_r \approx \omega_{q'} = (q'M + \mu + \nu_\beta + m\nu_s)\omega_0 \;. \end{cases} \tag{11.6}$$

Similar to the situation of longitudinal coupled-bunch instabilities, mode $\mu = 0$ and mode $\mu = \frac{1}{2}M$ if M is even receive contributions from both the positive-frequency side and negative-frequency side. In the language of only positive frequencies, there are the upper and lower betatron sidebands flanking each revolution harmonic line. The lower sideband originates from negative frequency and is therefore antidamped. For these two modes, both the upper and lower sidebands correspond to the same coupled-bunch mode. If the resonant frequency of the resonance leans more towards the lower sideband, there will be a growth. If the resonant frequency leans more towards the upper side band, there will be damping. This is the Robinson's stability analog in the transverse phase plane. However, sometimes it is not so easy to identify which is the lower sideband and which is the upper sideband. This is because the residual betatron tune $[\nu_\beta]$ or the noninteger part of the betatron tune can assume any value between 0 and 1. If $[\nu_\beta] > 0.5$, the upper betatron sideband of a harmonic will have a higher frequency than the lower betatron sideband of the next harmonic.

There is one important difference between transverse coupled-bunch instabilities driven by the resistive-wall impedance and by the higher-order resonant modes. The former is at very low frequency and therefore the form factor F_0' is close to 1 when the chromaticity is zero. The latter, however, is at the high frequencies of the resonances. The form factor usually assumes a much smaller value unless the bunch is very short and we sometimes refer this to "damping" from the spread of the bunch.

This instability can be observed easily in the frequency domain at the lower betatron sidebands flanking the harmonic lines. If a particular lower betatron sideband grows strongly, we subtract the betatron tune ν_β (not $[\nu_\beta]$) to find out which harmonic line it is associated with. Then from Eq. (11.6), we can determine which coupled-bunch mode μ it is. To damp this transverse coupled-bunch instability, one can identify the offending resonant modes in the cavities and damp them passively using an antenna. A tune spread due to the slip factor η or from an octupole can also contribute to the damping. When the above are not efficient enough, a transverse bunch-to-bunch damper will be required. If we can identify the annoying mode, a mode damper of narrow band will do the job. To damp couple-bunch instabilities without knowing the annoying mode, a wideband bunch-by-bunch damper is necessary.

Similar to longitudinal coupled-bunch instabilities, transverse coupled-bunch instabilities can also be damped by modulation coupling from an uneven fill in the ring discussed in Sec. 9.3.4.

11.3 Exercises

11.1. For the example of resistive-wall driven coupled-bunch instability of the Tevatron at the fixed target mode, try to sum up the contribution for all frequencies for the $\mu = 1093$ mode and compare the result of taking only the lowest frequency line.

11.2. For the same example in Exercise 11.1, compare the growth rates of mode $\mu = 1092,\ 1091,\ \cdots$, with mode 1093. How many modes do we need to include so that the growth rate drops to below $\frac{1}{4}$ of that of mode 1093?

11.3. For a narrow resonance that has a total width larger than $2[\nu_\beta]\omega_0$ where $[\nu_\beta]$ is the residual betatron tune and the bunch power spectrum is much wider than the revolution frequency, show that the growth rate is given by

$$\frac{1}{\tau_{m\mu}} \approx \frac{eMI_bc}{4\pi\nu_\beta E_0}\,\frac{h_m(\omega_r-\chi/\tau_L)}{B\sum_{q'} h_m(\omega_{q'}-\chi/\tau_L)}\times$$
$$\times\left\{\mathcal{R}e\, Z_1^\perp[(q_1M-\mu-\nu_\beta)\omega_0-m\omega_s] - \mathcal{R}e\, Z_1^\perp[(q_2M+\mu+\nu_\beta)\omega_0+m\omega_s]\right\}\ , \qquad (11.7)$$

where q_1 and q_2 are some positive integer so that

$$\begin{aligned}(q_1M-\mu-\nu_\beta)\omega_0 &\approx \omega_r\ ,\\ (q_2M+\mu+\nu_\beta)\omega_0 &\approx \omega_r\ .\end{aligned} \qquad (11.8)$$

Such q_1 and q_2 are possible only when $\mu = 0$ or $\mu = M/2$ if M is even. Therefore whether the coupled-bunch mode is stable or unstable depends on whether the resonance is leaning more towards the upper betatron sideband or the lower betatron sideband.

Bibliography

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