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# PHYSICS OF INTENSITY DEPENDENT BEAM INSTABILITIES

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This is a revised edition of the lecture notes, *Physics of Collective Beam Instabilities*, given at the US Particle Accelerator School, SUNY Stony Brook, New York, June 5-16, 2000. Many new additions have been made and many errors corrected.

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